PERMUTATION AND COMBINATION

1. **Factorial Notation:** The continued product of the first n natural numbers is defined by the symbol $\underline{|n}$ or $n!$ and is read as 'Factorial n'.

   $n! = 1.\,2.\,3.\,...\,n$

   $n! = n\,(n-1)! = n\,(n-1)\,(n-2)!$

2. **The Sum Rule:** Suppose a work A can occur in m ways and B can occur in n ways and both cannot occur simultaneously. Then A or B (at least one of them) can occur in $(m + n)$ ways. This rule is also applicable for two or more exclusive events.

3. **The Product Rule:** Suppose there are two works A and B. Let A can occur in m ways and B in n ways. Suppose that the ways for A and B are not related in the sense that B

occur in n ways regardless the outcome of A, then both A and B occur in mn ways.

For example, let there are two questions A and B which can be solved by two methods and 3 methods respectively. Then, A or B can be solved in $2 + 3 = 5$ ways and boh A and B in $2 \times 3 = 6$ ways.

4. **Permutation:** Each of the arrangements that can be made by taking some or all of a number of dissimilar things (objects) is called *Permutation*.
5. **Combination:** Each of the different group or selection which can be made by taking some or all of a number of things (irrespective of order) is called a *combination*.

   Derangements: Any change in the given order of the things is called a derangements.
6. The number of permutations of n distinct things taken all at a time = $n!$
7. The number of permutations of n dissimilar things taken r at a time when each thing can be repeated any number of times = n^r.
8. Number of permutations of n distinct things taken r at a time, $0 \leq r \leq n$

$$= n\,(n-1)\,(n-2)\,...\,(n-r+1)$$
$$= n!/(n-r)! = {}^nP_r.$$

This is equivalent to filling r places by r objects taking from n distinct objects.

9. The number of permutations of n distinct things taken r at a time when p particulars things always occur $= {}^{(n-p)}C_{(r-p)} \cdot r!$.

10. Number of permutations (arrangements) of n distinct things taken r at a time when p particular things never occur

$$= {}^{(n-p)}C_r \cdot r!$$

11. Number of permutations of n things, taken all at a time when p_1 are alike of one kind, p_2 are alike of second kind, ... p_r of them are alike of the r[th] kind $p_1 + p_2 + ... + p_r \leq n$, and remaining things are all different $= n!/\{p_1!\, p_2! \dots p_r!\}$.

12. The number of combinations of n objects taken r at a time, $0 \leq r \leq n$.

$$= {}^{n}C_r = \frac{n!}{(n-r)!r!}$$

13. The number of combinations of n distinct objects taken r at a time when any object may be repeated any number of times.

$$= \text{coeffcient of } x^r \text{ in } (1 + x + x^2 + x^3 + \dots + x^r)^n$$

$$= \text{coefficient of } x^r \text{ in } (1-x)^{-n} = {}^{n+r-1}C_r$$

14. Number of combinations of n distinct things taken r at a time when p particular things always occur

$$= {}^{(n-p)}C_{(r-p)}$$

15. Number of combinations of n distinct things taken r at a time when p particular things never occur $= {}^{(n-p)}C_r$.

16. The number of combinations if some or all n things be taken at a time

$$= {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1.$$

17. If there are p_1 objects of one kind, p_2 objects of second kind, ..., p_n objects of n^{th} kind, then the number of ways of choosing r objects out of these $(p_1 + p_2 + \ldots + p_n)$ objects

$$= \text{coefficient of } x^r \text{ in } \left(1 + x + \ldots + x^{p_1}\right)$$

$$\left(1 + x + \ldots + x^{p_2}\right) \ldots \left(1 + x + \ldots x^{p_n}\right)$$

If one object of each kind is to be included in such a collection then the number of ways of choosing r objects.

$$= \text{coefficient of } x^r \text{ in the product}$$

$$\left(x + x^2 + \ldots + x^{p_1}\right) \left(x + x^2 + \ldots + x^{p_2}\right) \ldots$$

$$\left(x + x^2 + \ldots + x^{p_n}\right).$$

18. Total number of ways to make a selection by taking some or all of $p_1 + p_2 + \dots p_r$ things where p_1 are alike of one kind, p_2 are alike of second kind, ..., p_r are alike of r^{th} kind is

$$= (p_1 + 1)(p_2 + 1)\dots(p_r + 1) - 1$$

19. The number of ways in which n distinct objects can be split into three groups containing respectively r, s and t objects, r, s and t are distinct and $r + s + t = n$, is given by

$$^{n}C_{r}\ ^{n-r}C_{s}\ ^{n-r-s}C_{t} = \frac{n!}{r!.s!.t!}$$

20. If 3n things are to be divided into the three equal groups, then the number of ways

$$= \frac{(3n)!}{n!.n!.n!.3!}$$

21. If $3n$ things are to be divided equally between 3 persons (*i.e.*, division of $3n$ things into 3 equal groups with permutation of groups),

then the number of ways $= \dfrac{(3n)!}{(n!)^3}$

22. If n things form an arrangement in a row, then the number of ways in which they can be deranged so that no one of them occupies its original place is

$$n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+(-1)^n.\frac{1}{n!}\right)$$

23. The number of circular permutations of n different things taken all at a time $= (n-1)!$

24. The number of arrangements of n persons on a round table $= (n-1)!$

25. The number of arrangement of n flowers to make a garland $= \frac{1}{2}\ (n-1)!$

26. ${}^nC_r = {}^nC_{n-r}$

27. ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

28. ${}^nC_r = {}^nC_s \Rightarrow r = s$ or $r + s = n$.

29. Greatest Value of ${}^nC_r$

${}^nC_r$ is greatest when $r = n/2$ if n is even.

$r = (n-1)/2$ or $(n+1)/2$ if n is odd.

30. Distinction between problems on "permutation and combination"

(*a*) In problems on arrangements, ordering, permutations, arranging, line up, we use the formula for Permutations.

(b) In problems on selection, subset, committee, groups, choice, we use the formula for Combinations.

(c) When to multiply and when to add, Multiply when there is 'AND' in the problem and 'Add' when there is 'OR'.